

Sheet 1.

1 Find $e(0)$ and $e(10)$ for

$$E(z) = \frac{1}{(z-1)(z-0.3)}$$

using Partial Fraction

Then check $e(0)$ using Initial value

Solⁿ

using Partial Fraction

$$E(z) = \frac{A_1}{z-1} + \frac{A_2}{z-0.3} = z^{-1} \left[\frac{A_1 z}{z-1} + \frac{A_2 z}{z-0.3} \right]$$

By inspection

$$A_1 = 1/0.7 = 1.43, \quad A_2 = 1/-0.7 = -1.43$$

$$\therefore E(z) = z^{-1} \left[\frac{1.43z}{z-1} - \frac{1.43z}{z-0.3} \right]$$

$z^{-1} \rightarrow$

$$e(k) = 1.43 u(k-1) - 1.43 (0.3)^{k-1} u(k-1)$$

$$\therefore e(0) = 1.43 u(-1) - 1.43 (0.3)^{-1} u(-1) = \text{Zero}$$

$$e(10) = 1.43 u(9) - 1.43 (0.3)^9 u(9) = 1.4286$$

Check of $e(0)$ using initial value

$$\begin{aligned} e(0) &= \lim_{b \rightarrow 0} e(b) = \lim_{z \rightarrow \infty} E(z) \\ &= \lim_{z \rightarrow \infty} \frac{1}{(z-1)(z-0.3)} = \frac{1}{\infty} = \text{Zero} \end{aligned}$$

2 Given: $e(t) = A \cos(\omega t)$, $E(z) = \frac{3z(z-0.6967)}{z^2 - 1.3934z + 1}$

Required: (with $T=0.2s$) \rightarrow Find A, ω

Solⁿ

$$e(kT) = A \cos(\omega kT)$$

from the given $E(z)$

$$\therefore \boxed{A = 3}$$

$$\therefore \cos(\omega T) = 0.6967 \rightarrow$$

$$E(z) = \frac{A z (z - \cos(\omega T))}{z^2 - 2z \cos(\omega T) + 1}$$

$$\omega = \frac{\cos^{-1}(0.6967)}{T} \quad 0.8$$

using radians

$$\therefore \omega = \frac{0.8}{0.2} = \boxed{4 \text{ rad/s}}$$

16] Given $y(k) - y(k-1) + y(k-2) = u(k)$

where $u(k) = 1$ for $k \geq 0$, $T=1s$

- Required:
- i) Find $y(0), y(1), y(2)$ and $y(3)$ using Power method
 - ii) Verify Part i by solving difference equation directly
 - iii) Solve $y(k)$ using Z-transform

solⁿ

$$y(k) - y(k-1) + y(k-2) = u(k)$$

taking Z-transform

$$Y(z) - z^{-1}Y(z) + z^{-2}Y(z) = \frac{1}{1-z^{-1}}$$

$$Y(z)(1 - z^{-1} + z^{-2}) = \frac{z}{z-1}$$

$$Y(z) = \frac{z}{(z-1)(1-z^{-1}+z^{-2})} = \frac{z^3}{(z-1)(z^2-z+1)}$$

i) Using power method (Long division)

$$\begin{array}{r} 1 + 2z^{-1} + 2z^{-2} + z^{-3} + z^{-6} \\ z^3 \overline{) z^3 - 2z^2 + 2z - 1} \end{array}$$

$$2z^2 - 2z + 1$$

$$2z^2 - 4z + 4 - 2z^{-1}$$

$$2z - 3 + 2z^{-1}$$

$$2z - 4 + 4z^{-1} - 2z^{-2}$$

$$1 - 2z^{-1} + 2z^{-2}$$

$$1 - 2z^{-1} + 2z^{-2} - z^{-3}$$

$$y(0) = 1$$

$$y(1) = 2$$

$$y(2) = 2$$

$$y(3) = 1$$

$$y(4) = 0$$

$$y(5) = 0$$

$$y(6) = 1$$

Note

for $Y(z) = 1 + 2z^{-1} +$

$$2z^{-2} + z^{-3} + \dots$$

knowing that

$$Y(z) = \sum_{k=0}^{\infty} y(k)z^{-k}$$

$$\therefore Y(z) = y(0) + y(1)z^{-1} + y(2)z^{-2} + \dots$$

$$z^{-3}$$

$$z^{-3} - 2z^{-4} + 2z^{-5} - z^{-6}$$

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$$ii) \quad y(k) - y(k-1) + y(k-2) = u(k)$$

$$\therefore y(k) = y(k-1) - y(k-2) + u(k)$$

$$\text{For } k=0 \longrightarrow y(0) = \cancel{y(-1)} - \cancel{y(-2)} + u(0) = 1$$

$$\text{For } k=1 \longrightarrow y(1) = y(0) - \cancel{y(-1)} + u(1) = 2$$

$$\text{For } k=2 \longrightarrow y(2) = y(1) - y(0) + u(2) = 2$$

$$\text{For } k=3 \longrightarrow y(3) = y(2) - y(1) + u(3) = 1$$

$$\text{For } k=4 \longrightarrow y(4) = y(3) - y(2) + u(4) = 0$$

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iii) Using Z-transform

$$Y(z) = \frac{Z^3}{(Z-1)(Z^2-Z+1)} = Z \left[\frac{Z^2}{(Z-1)(Z^2-Z+1)} \right]$$

$$= Z \left[\frac{A}{Z-1} + \frac{BZ+C}{Z^2-Z+1} \right] = \frac{Z}{Z-1} + \frac{Z}{Z^2-Z+1}$$

we know that

$$Z \left[\sin(ak) \right]_{r=1} = \frac{Z \sin(a)}{Z^2 - 2Z \cos(a) + 1}$$

$$\therefore Y(z) = \frac{Z}{Z-1} + \frac{Z \sin(a)}{(Z^2 - Z + 1) \sin(a)} \quad \left(\text{we multiplied by } \frac{\sin(a)}{\sin(a)} \right)$$

$$\text{By analogy: } 2\cos a = 1 \longrightarrow \cos a = 0.5 \longrightarrow a = \pi/3 \text{ rad}$$

$$\therefore \sin a = \sin\left(\frac{\pi}{3}\right) = \sqrt{3}/2$$

$$\therefore Y(z) = \frac{Z}{Z-1} + \frac{2}{\sqrt{3}} \frac{Z \sin(a)}{Z^2 - 2\cos(a)Z + 1}$$

$$\therefore \boxed{y(k) = u(k) + \frac{2}{\sqrt{3}} \sin\left(\frac{\pi}{3}k\right)}$$

check

$$y(0) = 1, \quad y(1) = 1 + \frac{2}{\sqrt{3}} \sin\left(\frac{\pi}{3}\right) = 2$$

$$y(2) = 1 + \frac{2}{\sqrt{3}} \sin\left(\frac{2\pi}{3}\right) = 2$$

$$y(3) = 1 + \frac{2}{\sqrt{3}} \sin(\pi) = 1$$